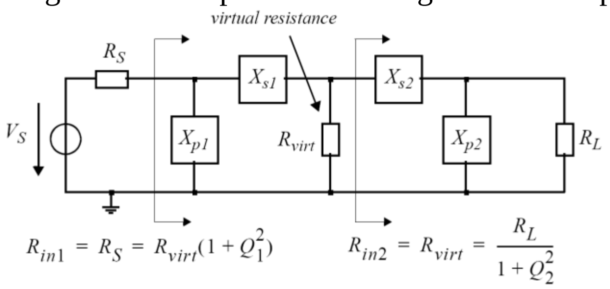
Serie 5: Π-Impedance Matching Circuit - Chapter 3

The Π-Impedance Matching Circuit is represented on Fig. 3.15 of Chap. 3 of the course book:



The Q_1 factor and the Q_2 factor are defined by the following relations:

$$Q_1 = \frac{R_S}{|Xp1|} \text{ and } Q_2 = \frac{R_L}{|Xp2|}$$

For remembrance, the quality factor of the Π-Impedance Matching Circuit is equal to :

$$Q_\Pi = \text{Max}\,(Q_1\,,\,Q_2).$$

By definition, R_{virt} corresponds to the resistance seen from the middle point as represented above.

Question 1:
Explain why the nature of the impedance X_{s2} and X_{p2} are opposite; that means that (X_{s2} is an inductor if X_{p2} is a capacitor) or (X_{s2} is a capacitor if X_{p2} is an inductor) if a resistance R_{virt} is seen from the middle point.

Question 2:
Explain why the nature of the impedance X_{s1} and X_{p1} are opposite; that means that (X_{s1} is an inductor if X_{p1} is a capacitor) or (X_{s1} is a capacitor if X_{p1} is an inductor) if a resistance R_{virt} is seen from the middle point.

Question 3:
Prove that R_{virt} can be expressed by :$R_{virt} = \frac{R_S}{1+Q_1^2}$ and $R_{virt} = \frac{R_L}{1+Q_2^2}$

Question 4:
Prove that Q_1 and Q_2 can be expressed by : $Q_1 = \frac{|X_{s1}|}{R_{virt}}$and $Q_2 = \frac{|X_{s2}|}{R_{virt}}$

Question 5:
Express the relations which allow to determine the expressions of $|X_{p1}|$,$|X_{s1}|$,$|X_{s2}|$ and $|X_{p2}|$ as a function of Q_Π , R_S and R_L.

1. **Explain why the nature of the impedance X_{s2} and X_{p2} are opposite; that means that (X_{s2} is an inductor if X_{p2} is a capacitor) or (X_{s2} is a capacitor if X_{p2} is an inductor) if a resistance R_{virt} is seen from the middle point.**

Answer:

The virtual resistance is a mathematical intermediate step which is used to simplify the calculations as follows. By assuming a resistive impedance between the two parts of the circuit, we can split it in two parts and calculate each one independently.

2. **Explain why the nature of the impedance X_{s1} and X_{p1} are opposite; that means that (X_{s1} is an inductor if X_{p1} is a capacitor) or (X_{s1} is a capacitor if X_{p1} is an inductor) if a resistance R_{virt} is seen from the middle point.**

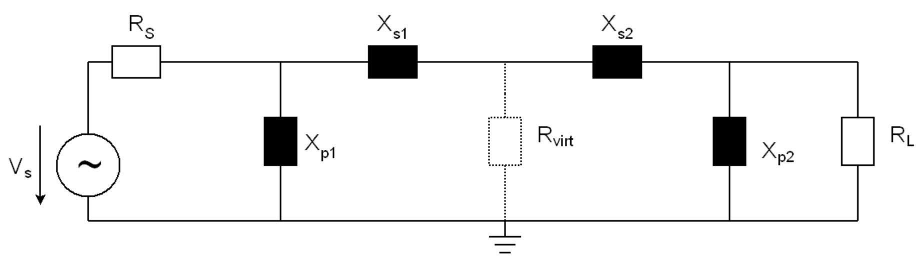
Answer:

It is exactly the same answer as in question 1).

3. **Prove that R_{virt} can be expressed by: $R_{virt} = \frac{R_s}{1+Q_1^2}$ and $R_{virt} = \frac{R_L}{1+Q_2^2}$**

Answer:

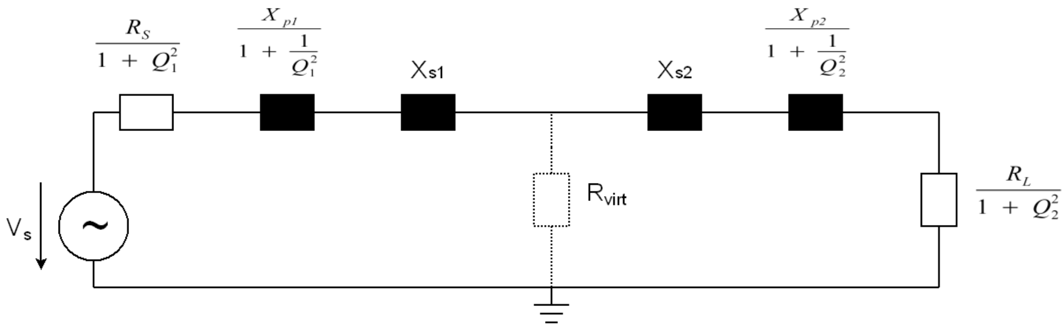
The matching circuit is given below. We first remove the shunt branches in order to simplify the circuit.



Since R_s and R_L make each a shunt circuit, their quality factors are given by :

$$Q_1 = \frac{R_S}{|X_{p1}|} \text{ and } Q_2 = \frac{R_L}{|X_{p2}|}$$

So, independently of their respective nature (capacitive or inductive), the shunt branches can be transformed each in a series branch in the same way as follows :



Because of impedance matching conditions, we have :

$$\frac{R_s}{1+Q_1^2} = R_{virt} = \frac{R_L}{1+Q_2^2}$$

and

$$\frac{X_{p1}}{1+\frac{1}{Q_1^2}} = -X_{s1} \text{ and } \frac{X_{p2}}{1+\frac{1}{Q_2^2}} = -X_{s2}$$

The first relation proves the equivalence.

4. **Prove that Q_1 and Q_2 can be expressed by: $Q_1 = \frac{|X_{s1}|}{R_{virt}}$ and $Q_2 = \frac{|X_{s2}|}{R_{virt}}$**

Answer:

As calculated previously, we have :

$$\frac{X_{p1}}{1+\frac{1}{Q_1^2}} = -X_{s1} \Rightarrow |X_{s1}| = \frac{|X_{p1}|}{1+\frac{1}{Q_1^2}} = \frac{Q_1^2 \cdot |X_{p1}|}{1+Q_1^2}$$

$$\frac{X_{p2}}{1+\frac{1}{Q_2^2}} = -X_{s2} \Rightarrow |X_{s2}| = \frac{|X_{p2}|}{1+\frac{1}{Q_2^2}} = \frac{Q_2^2 \cdot |X_{p2}|}{1+Q_2^2}$$

We can thus write :

$$\frac{|X_{s1}|}{R_{virt}} = \frac{Q_1^2 \cdot |X_{p1}|}{1+Q_1^2} \cdot \frac{1+Q_1^2}{R_s} = Q_1^2 \cdot \frac{|X_{p1}|}{R_s} = Q_1^2 \cdot \frac{1}{Q_1} = Q_1$$

$$\frac{|X_{s2}|}{R_{virt}} = \frac{Q_2^2 \cdot |X_{p2}|}{1+Q_2^2} \cdot \frac{1+Q_2^2}{R_L} = Q_2^2 \cdot \frac{|X_{p2}|}{R_L} = Q_2^2 \cdot \frac{1}{Q_2} = Q_2$$

This proves the equivalence.

5. Express the relations which allow to determine the expressions of $|X_{p1}|$,$|X_{s1}|$,$|X_{s2}|$ and $|X_{p2}|$ as a function of QΠ , RS and RL.

Answer:

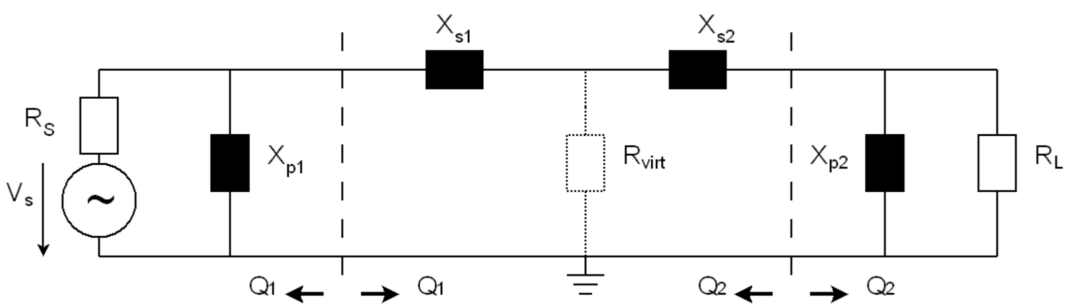
According to relation (3.6) on page 3.14 of the course, we know that the Q parameter of the Pi matching network is expressed by:

$$Q_{Pi} = \sqrt{\frac{Max(R_{s,}, R_L)}{R_{virt}} - 1}$$

Therefore:

$$R_{virt} = \frac{Max(R_{s,}, R_L)}{1+Q_{Pi}^2}$$

A useful method for remembering all these equations is given below by considering the circuit shown below where all reactive part of the original circuit can be coupled to a resistive one which gives for every couple a quality factor.



It becomes thus easy to get the corresponding equation for the right side of the above circuit:

$$Q_2 = \frac{R_L}{|X_{p2}|}$$

This relation implies that:

$$|X_{p2}| = \frac{R_L}{Q_2}$$

with:

$$Q_2 = \sqrt{\frac{R_L}{R_{virt}} - 1}$$

$$R_{virt} = \frac{Max(R_{s,}, R_L)}{1 + Q_{Pi}^2}$$

Therefore, we can express $|X_{p2}|$ as a function of Q_{pi} , R_S and R_L.

The same method applies for $|X_{s2}|$ as a function of Q_{pi} , R_S and R_L because:

$$Q_2 = \frac{|X_{s2}|}{R_{virt}}$$

This relation implies that:

$$|X_{s2}| = Q_2 . R_{virt}$$

with:

$$Q_2 = \sqrt{\frac{R_L}{R_{virt}} - 1}$$

$$R_{virt} = \frac{Max(R_{s,}, R_L)}{1 + Q_{Pi}^2}$$

Therefore, we can express $|X_{s2}|$ as a function of Q_{pi} , R_S and R_L.

Through the same method, it is easy to get the corresponding equation for the left side of the same circuit:

$$Q_1 = \frac{R_s}{|X_{p1}|}$$

This relation implies that:

$$|X_{p1}| = \frac{R_s}{Q_1}$$

with:

$$Q_1 = \sqrt{\frac{R_s}{R_{virt}} - 1}$$

$$R_{virt} = \frac{Max(R_{s,}\,,\, R_L)}{1 + Q_{Pi}^2}$$

Therefore, we can express $|X_{p1}|$ as a function of Q_{pi} , R_S and R_L.

The same method applies for $|X_{s1}|$ as a function of Q_{pi} , R_S and R_L because:

$$Q_1 = \frac{|X_{s1}|}{R_{virt}}$$

This relation implies that:

$$|X_{s1}| = Q_1 . R_{virt}$$

with:

$$Q_1 = \sqrt{\frac{R_s}{R_{virt}} - 1}$$

$$R_{virt} = \frac{Max(R_{s,}\,,\, R_L)}{1 + Q_{Pi}^2}$$

Therefore, we can express $|X_{s1}|$ as a function of Q_{pi} , R_S and R_L.